

Final Exam, MTH 211, Fall 2009

Ayman Badawi

QUESTION 1. 39 points, each = 3 points Write down T OR F (no justification is needed)

- (i) Each interior angle of a regular 10-gon is 36 degree.
- (ii) Using unmarked ruler and a compass only we can construct a regular 28-gon.
- (iii) Let D is a circle with O as the center, H be another circle inside D and has the same center as D , and let F be the inversion of H with respect to D . Then D lies inside F .
- (iv) If we constructed an angle α by unmarked ruler and a compass and $n = \frac{360}{\alpha}$ is a whole number, then we can construct a regular n -gon.
- (v) $a'b'$ is the inversion of the line segment ab with respect to C
- (vi) If the hyper-line L_1 is parallel to the hyper-line L_2 and the hyper-line L_1 is parallel to the hyper-line L_3 , then L_2 is parallel to L_3 .
- (vii) It is possible to construct a hyper-square so that the sum of all 4 interior angles equals to 360.
- (viii) if C is the golden cut of the line segment ab such that $ab/ac =$ the golden ratio, then $\frac{ab+cb}{ab}$ is still the golden ratio.
- (ix) consider the square $abcd$. Let us replace the side ab by an arc amb (see figure) .
If we reflect amb about the line EF and we removed the side cd , then we get a new object that can be used to tile a plane.
- (x) It is possible to construct a regular 50-gon using unmarked ruler and a compass
- (xi) It is possible to construct a regular 42-gon using unmarked ruler and a compass
- (xii) In Fibonacci sequence F_n , we know that $F_1 = 1$, $F_2 = 1$, and $F_n = F_{n-1} + F_{n-2}$. Also we know that when n is so huge then the ratio of $\frac{F_{n+1}}{F_n} =$ golden ratio. Now let us assume that $F_1 = 1/2$, $F_2 = 1/2$, and $F_n = F_{n-1} + F_{n-2}$ (so $F_3 = F_2 + F_1 = 1$, $F_4 = F_3 + F_2 = 1.5$, $F_5 = F_4 + F_3 = 2$, and so on ...). Then when n becomes huge the ratio of $\frac{F_{n+1}}{F_n} = \frac{1}{2}$ of the golden ratio.
- (xiii) This is MTH 221, you are in Chemistry 109, and your instructor name is Aman Badawi

QUESTION 2. 15 points Let C be a circle with radius 3, center O , and A be a point inside C such that $d(O, A) = 1\text{ cm}$ (the Euclidean distance from O to A is 1). Let D be a circle passes through A and orthogonal to C (the two circles make 90 degrees, I mean D is perpendicular to C). Show that the radius of D must be greater or equal to 4. I mean use simple math to verify that. After you are done with the verifications, state the steps that you will use to construct such circle D with radius 5.

QUESTION 3. 16 points a.) USE UNMARKED RULER and a COMPASS ONLY. Given two lines intersect at a point O and A is a point that does not lie on any of the two lines, construct a line passing through A and intersecting the two lines at the points B and C in such a way that $AB = AC$. State the steps of construction. No math justification is needed.

b) Construct a 2-points perspective image of a rectangle that is not a square. State the steps of construction without math justification.

QUESTION 4. 15 points Let H be the horizon circle with radius 4 and center O . Let A, B be two points inside H such that they do not lie on any diameter of H . Given $d(O, A) = d(O, B) = 2\text{cm}$. Construct a hyper line, SAY L , that passes through A , and B . a) Show the steps of construction.

b) Now choose two points, F, D inside H such that F, D lie on L too. Explain in at most two lines why do C_A, C_B, C_D, C_F intersect exactly in one point.

c) Use a marked ruler to find $d_h(A, B)$.

QUESTION 5. 15 points

a triangle abc is called semi-acute triangle if $ab = ac$ and $ab/bc = \frac{1}{2}$ of the golden ratio. Now you have a thin wire that has length 12cm. Divide the wire into 6 pieces to make two semi-acute triangles. Show the steps of construction. Then calculate the angles of such triangle.

Faculty information

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